Solutions of Problem 1 : Human Eye

First Part

1. As the object is located at infinity, the focal distance f of the lens is f=D. The focal power is 1/D = 41.7 Dpt.
2. An object is located at L=1m
   a. The focal distance of the lens should be $f = \frac{LD}{L+D}$ =23mm
   b. It depends how we calculate. Actually, the original question was, "what is the MAXIMAL size", but due to typing error maximal transformed into minimal. That's why, we will treat the question with the minimal size, and the note will take into account the presented text, i.e the minimal size. For sake of clarity we will give also the answer of the maximal size. Now, let's first try to calculate what is the mean distance between two points, knowing the density of the points. First, let's suppose that the cells are homogeneous discs, and that at the maximal density point those discs are compactly stacked. So, what is the form of this stack? Each disc has the same radius d/2. This means that the first nearest discs are located around a circle with radius d. But those discs are in contact with each other. This means that the triangle formed by the centres of 3 touching each other discs has three identical sides. This means that the centres of the discs form a hexagonal mesh. Now, the elementary cell of the hexagonal mesh is a parallelogram with 4 identical sides equal to d. Its surface is therefore $S=d^2\cos(30°)$. The surface occupied by the spheres in the parallelogram is $S' = \frac{2d^2}{4}\frac{\pi}{6} + 2\frac{d^2}{4}\frac{2\pi}{6} = \frac{\pi}{4}d^2$. The fill factor is then $F = \frac{S'}{S} = \frac{\pi}{4\cos(30°)}$. Now, let's take a general surface Φ much bigger than the surface of our discs. Then, the number of compactly stacked discs that we could put in the surface can be approximated by $N = \frac{\Phi}{\pi d^2/4}F = \Phi \big/ d^2\cos(30°)$. Therefore, the density of the discs is $\rho = \frac{N}{\Phi} = \frac{1}{d^2\cos(30°)}$. This gives us that the mean distance between two discs is $d = \sqrt{1 \big/ \rho\cos(30°)}$. Now lets R be the minimal size of the object. By simple geometry we have $R/r = L/D$ and $R = \sqrt{1/\rho\cos(30°)}\,\frac{L}{D}$ =116µm.

      Now, if we want to calculate the maximal size, it will be given by the angular dependency of the cons. We now that out of 20° of the visual axe, no more cones exist. So the maximal size is L.tan(20°)=36cm. Let's comment on this size. We know that we can see much bigger objects situated at 1m. But only within a circular region of r=36cm we could clearly see the objects, and out of that region, we will need to turn the eyes in order to see clearly.
3. Sight of a short seeing person.
   a. Lets' first calculate the focal distance of the short seeing eye. It is $f' = \frac{LD'}{L+D'}$ =22.83mm, and D'=D-1mm=23mm. Now lets calculate the distance of the virtual image to the eye if we want it to be clearly seen.

$L^{'} = \frac{Df^{'}}{D-f^{'}}$=46,83cm. No if *l=1cm* is the distance eye-glass, we obtain for the focal distance of the glass $f_1 = \frac{(L^{'}-l)(L-l)}{L^{'}-L}$=-54,13cm. The ” – “ sign means that the lens is divergent.

b. In the case of contact lenses *l=0* and $f_1 = \frac{L^{'}L}{L^{'}-L}$=-55,49

4. Eye resolution
   a. As ρ is a linear function of θ the mean value of the density is simply $\overline{\rho} = \frac{\rho(\theta_{\max}) - \rho(0)}{2}$=75000mm^{-1}. So the cell’s size is $r = \sqrt{\frac{1}{\overline{\rho}}\cos(30°)}$ =4μm
   b. By the result from 2.b $R = \sqrt{1/\overline{\rho}\cos(30°)}\frac{L}{D}$. More we have an object that is located far from the eye, bigger it should be to be clearly seen.
   c. Using 4 b R=163μm. This value is bigger that the previously calculated but is a good approximation of the resolution in the eye in the whole visible region and not only in its centre.
   d. The minimal visible angle α=arctan*(R/L)*=0.0093°. The angular resolution is 1/α.

Second Part

1. We have to calculate the angular resolution of the spherical eye as a function of the angle of the incident eye. In the clear part of the vision there are two types of cells, but the sticks has a lower response. We can model this by lowering their density by 2. So for the clear part of the vision we obtain fort the angular resolution $\alpha = \arctan\left(\frac{1}{\sqrt{(\rho_{cones}(\theta) + \frac{\rho_{stics}(\theta)}{2})\cos(30°)}}\frac{1}{D}\right)$. The angular resolution is 1/α.
2. The result is identical to 1. We have to see what happens when the object moves away from the visual axis. Well, arctan is an increasing function that means that if the argument of the function increase the function increases also. As the density of the cells in the human eye decrease, the argument increases and thus the resolution which is 1/α decreases.

Solutions of Problem 2

1. The Ohm’s law is : $U = I.R \Leftrightarrow \Delta P = Q.R^h$
2. The power of the hart is $W_c = \Delta P.Q$. Numerical application $W_c = 1.1W$
3. $N_k = n_1.n_2...n_k$
4. $Q_k = \pi r_k^2\overline{u}_k = n_{k+1}Q_{k+1} = n_{k+1}.\pi r_{k+1}^2\overline{u}_{k+1}$

5. $n_k = n$, $\bar{u}_k = u$ so $\pi r_k^2 = n\pi r_{k+1}^2 \Rightarrow \beta = n^{-1/2}$

6. $V_b = \sum_{k=0}^{k=N} \pi r_k^2 l_k N_k = \pi r_0^2 l_0 N_0 \sum_{k=0}^{k=N} (\beta^2 \gamma n)^k = V_0 \dfrac{1-(\beta^2\gamma n)^{N+1}}{1-\beta^2\gamma n}$ Finally for N>>1 and

   $\beta^2\gamma n < 1$ we find $V_b = \dfrac{V_0}{1-\beta^2\gamma n}$

7. $R_t = \sum_{k=0}^{k=N} \dfrac{R_k^h}{N_k} = \sum_{k=0}^{k=N} \dfrac{8\mu l_k}{\pi r_k^4 N_k} = R_0^h \sum_{k=0}^{k=N} \left(\dfrac{\gamma}{\beta^4 n}\right)^k = R_0^h \dfrac{1-\left(\dfrac{\gamma}{\beta^4 n}\right)^{N+1}}{1-\left(\dfrac{\gamma}{\beta^4 n}\right)}$ After the

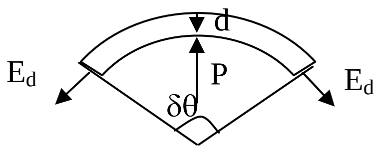
   approximations we find like in 6 $R_t = \dfrac{R_0^h}{1-\left(\dfrac{\gamma}{\beta^4 n}\right)}$

8. $Wc = \dfrac{Q^2 R_0^h}{1-\dfrac{\gamma}{\beta^4 n}}$

9. We minimize $F(n) = W_c + b.V_b$ making $F'(n) = 0$ leads to the following equations:
   a. $\beta^6 n^2 = 1$
   b. $\dfrac{\gamma}{\beta^4 n} = 1$
   c. $n\beta^2\gamma = 1$

We can easily check that $\beta = n^{-1/3}$ and $\gamma = n^{-1/3}$ are the unique solutions of those equations

10. We resolve on the following system (see figure)



$dE\delta\theta = \Pr\delta\theta \Rightarrow Ed = rP$

11. Following Ohm's law we find $\Delta P = \dfrac{8\mu l}{\pi (Ed)^4} P^4 Q$

12. Writing this equation in differential form we find: $dP = \dfrac{8\mu}{\pi (Ed)^4} P^4 Q.dl$ After

    integration we find $P^{-3} = P_0^{-3} + \dfrac{24\mu l Q}{\pi (Ed)^4}$